Robust Image Transmission Over Energy-Constrained Time-Varying Channels Using Multiresolution Joint Source–Channel Coding

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***Abstract*— We explore joint source-channel coding (JSCC) for time-varying channels using a multiresolution framework for both source coding and transmission via novel multiresolution modulation constellations. We consider the problem of still image transmission over time-varying channels with the channel state information (CSI) available at 1) receiver only and 2) both transmitter and receiver being informed about the state of the channel, and we quantify the effect of CSI availability on the performance. Our source model is based on the wavelet image decomposition, which generates a collection of subbands modeled by the family of generalized Gaussian distributions. We describe an algorithm that jointly optimizes the design of the multiresolution source codebook, the multiresolution constellation, and the decoding strategy of optimally matching the source resolution and signal constellation resolution "trees" in accordance with the time-varying channel and show how this leads to improved performance over existing methods. The real-time operation needs only table lookups. Our results based on a wavelet image representation show that our multiresolution-based optimized system attains gains on the order of 2 dB in the reconstructed image quality over single-resolution systems using channel optimized source coding.**

***Index Terms*—Data communication, hierarchical systems, image communication, quantization, source coding.**

I. Introduction and Background

IN MODERN practical communication systems, the source coder is in most cases separate from the channel coder both physically and conceptually. This approach can be potentially very inefficient for applications involving the transmission of video and images. This fact is already understood and generally acknowledged by researchers [1]–[4], but efficient methods of combining source and channel coders into a joint system are still under investigation. The increased complexity of the problem involving this joint design calls for the formulation of new frameworks that expose the fundamental underlying tradeoffs in source and channel coding. In this paper, we introduce a multiresolution-based framework to combine source and channel coding, and show how this approach leads to significant improvements in end-to-end system performance over conventional methods. Note that in contrast to the traditional usage of the term "multiresolution" in the source coding community, we will use this term in the paper in the context of both source coding (to refer to a hierarchy of codebook resolutions) and channel coding (to refer to a hierarchy of modulation constellations).

The benefits of multiresolution joint source-channel coding (JSCC) for digital broadcast were established recently in [1]. The fundamental idea there is derived from Cover's classic result [5] that in multiuser communications, where a single source transmits information to two (or more) receivers of different fidelity, joint information transfer is efficient if the transmitter superimposes the information meant for the strong receiver in the information meant for the weaker one; see Fig. 1. This endorses the use of a multiresolution framework, where both receivers have access to "coarse" information, while enabling the stronger receiver to extract the underlying "detail" information as well. In [1], this idea was applied to the design of modulation constellations; see Fig. 3 for an example of two-level multiresolution constellations, which can be seen to contain "clouds" of "satellites," characterized by an intracloud to intercloud distance ratio μ. Note that two levels of unequal noise immunity are offered by these constellations, as represented by the satellites and the clouds in which they are embedded. This multiresolution transmission scheme can be matched to an multiresolution source-coding scheme, e.g., using hierarchical subband coding (see references in [6] for details), where the coarse (important) source layer maps to the clouds, and the detail (refinement) layer maps to satellites within each cloud.

In this work, we describe how the use of multiresolution constellations for point-to-point slow time-varying channels can lead to similar gains as for the broadcast channel. (The analogy is not surprising since both cases refer to multichannel environments.) We present an optimization algorithm for multiresolution JSCC and explain its application with several illustrative examples. In seeking a multiresolution environment, we are motivated by the increasing need to be *flexible* and *scalable* in source and channel coding architectures due to increased demands for interconnectivity and heterogeneity. We consider a flat-fading time-varying channel, which is a popular model in wireless communications, with interleaving

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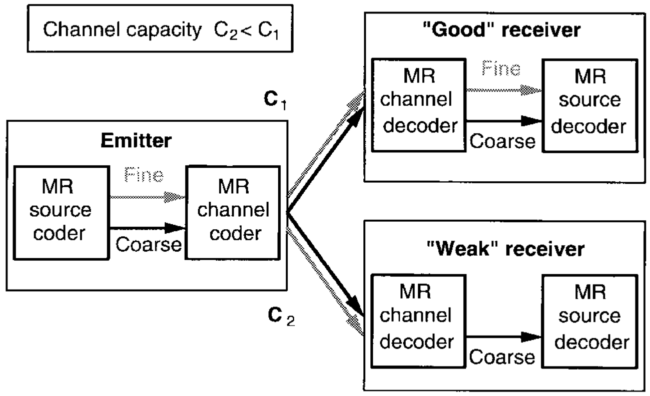


Fig. 1. Block diagram of two-resolution joint source channel coding scheme.

being used to render the channel memoryless. In this work, we tackle both important cases 1) when channel state information (CSI) indicating the instantaneous channel carrier-to-noise ratio (CNR)[1] is available only at the receiver, as applicable when there is no feedback channel from the receiver to the transmitter and 2) when CSI is available at both the receiver and transmitter. An added attraction of our framework is that it enables us to quantify the performance loss between cases 1) and 2). The multiresolution framework is advocated for case 1), whereas it is not needed for case 2). For typical scenarios, as considered in this paper, we show that there is little performance loss between cases 1) and 2), validating the choice of a multiresolution framework for case 1).

In motivating our work, we start with a brief overview. We begin by noting that optimal source coding has been a field of considerable interest and is well understood, e.g., the Lloyd–Max algorithm for scalar quantizer design and the generalized Lloyd algorithm (GLA) for vector quantizer design [7]. These algorithms optimize the source codebook design using essentially coordinate-descent type optimization methods consisting of alternate iterative optimizations of the encoder (with the decoder being fixed) and the decoder (with the encoder being fixed) and assuming a noiseless transmission channel. This design methodology has been extended to the case of noisy channels by several researchers (e.g., [2], [8], [9], and others) assuming channel models like the binary-symmetric-channel (BSC) or more general multialphabet channels with a given (i.e., *fixed*) transition probability matrix relating the channel input/output alphabets.

Finite-state noisy time-varying channels were considered previously in [10] and [11], where iterative algorithms to design the source coder were proposed. Additionally, the impact of errors in the channel state information was addressed in [11]. The setups in [10] and [11], however, assume a BSC model for each channel state (which is equivalently derived from a continuous channel with a binary modulation format), leaving no room to explore larger constellations or their optimized design. An illustration of the usefulness of allowing larger alphabet constellations (in addition to providing better bandwidth utilization) involves the special case of a Gaussian source transmitted over an AWGN channel. In [12, p. 162], Berger showed that the best transmission system for this case uses simple linear analog amplitude modulation to achieve the optimal performance.[2] Such a system can be approximated arbitrary closely with an N-PAM digital constellation, as used in this paper, as N gets sufficiently large.

All the approaches mentioned above essentially treat the channel as a "black box" with no control over transitional error probabilities (time-varying or time-invariant). This raises the interesting question of whether further gains can be attained by having the freedom to optimally alter these symbol transition error probabilities (by including the modulation constellation in the loop) and judiciously trading off some error probabilities for others *while keeping the transmitted modulation energy/bandwidth fixed.* This leads to the problem of jointly designing the source codebook and the channel constellation within the constraints of fixed transmission energy in order to minimize the overall end-to-end distortion. This has been addressed recently in [14], where a single resolution design was used, the transmitter was assumed to be informed of the CSI, and the optimized modulation constellation was unstructured.

In this paper, in addition to tackling the JSCC problem in a more intuitively appealing framework, we depart in three significant regards.

1) We use a *multiresolution approach* advocating the use of multiple codebook resolutions.
2) In the interests of practicality, we impose a *regular structure* on the resulting optimized constellation (we address the inefficiency due to this by giving an upper bound for a special case) using the intuitive idea of clouds and satellites (see Fig. 3).
3) We use a multistate additive white Gaussian noise (AWGN) channel model that can handle any type of CNR distribution for memoryless channels (we will focus on the Rayleigh distribution in this paper).

In the interests of simplicity and clarity of presentation, we confine ourselves to a scalar quantization framework, although extensions to vector quantization can be made at increased complexity.

The main contribution of this paper is that it tackles the JSCC problem for time-varying channels in a multiresolution setting using multiresolution source codebooks and embedded channel constellations. We show how the optimal design strategy under the given practical constraints dictates for *the receiver to adjust its source codebook resolution according to the channel state information*, and we quantify how this should happen. In Section II, we describe our models and illustrate the multiresolution JSCC design. Section III treats the case of a single i.i.d. source transmitted by an uninformed transmitter over a multistate AWGN channel. We extend our study of a single i.i.d. source to the case of a composite source in Section IV. In Section V, we discuss the case of the informed

[1]Note that we will use CNR to refer to the channel carrier-to-noise ratio, while we will use SSNR to refer to the source signal-to-noise ratio [7].

[2]Note that for this special case, this is a remarkable result, considering that in general, one has to resort to the infinitely complex, random-coding arguments of Shannon to attain the theoretical bound [13].

transmitter and, finally, conclude with some discussions in Section VI.

II. Problem Formulation and Proposed Framework

A. Communication Channels

In this work, we assume time-varying channels that we characterize by a finite (but large) number of states with a known probability of occurrence, i.e., the long-term statistics are assumed available *a priori* at both receiver and transmitter, even for the uninformed transmitter case. We model each particular channel state as an AWGN channel with a distinct noise variance. Although the model may appear to be restrictive, it allows us to approximate a wide variety of realistic channels such as flat fading channels or multicarrier channels while maintaining a moderate complexity of the analysis. Our channel model is similar to the one suggested in [15]: It is a special case of it in some regards but a generalization of it in other regards. It is a simplified version of [15] in the sense that we assume a perfect channel state information. On the other hand, by assuming a more general AWGN model, we have direct control over the modem parameters, and we can optimize the signal constellation over a larger than binary (as is effectively done in [15]) alphabets. To illustrate our model, consider the following example. Suppose we desire to approximate a memoryless Rayleigh channel by a multistate AWGN channel. By letting $h^2 = E/N_0$ be the received signal-to-noise power ratio, we make use of the fact that the instantaneous h-parameter has the Rayleigh probability density function (pdf)

$$f(h) = 2h/h_0^2 \exp(-h^2/h_0^2)$$

where $h_0 = \dfrac{2}{\sqrt{\pi}} \mathcal{E}\{h\}$ (with $\mathcal{E}\{\}$ denoting expectation).

By approximating this distribution with a piecewise constant pdf having N regions, we achieve a multistate AWGN representation that becomes exact as N becomes large (see Fig. 2). This approximation can be made for other CNR distributions as well, as long as the channel is memoryless. Note that our model is valid for frequency-varying channels (i.e., multicarrier channels) if the horizontal axis is used to represent "frequency" rather than "time." In addition, note that in practical applications, a similar "quantization" of channel characteristics is usually present. For example, power control algorithms in cellular communications usually allow the transmitted power to change in discrete steps (2 dB for GSM [16]) to compensate for varying channel conditions.

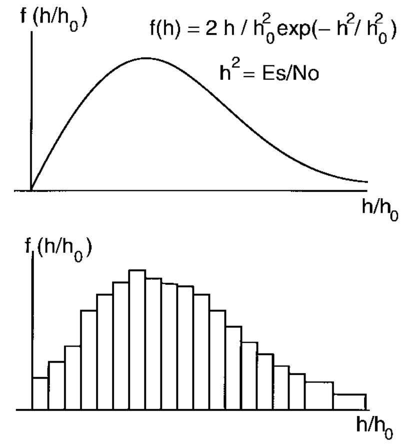


Fig. 2. Example of approximation of memoryless Rayleigh channel by a multistate AWGN channel model.

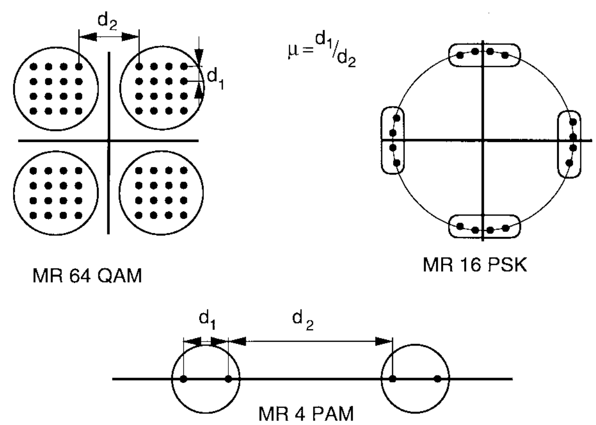


Fig. 3. Some multiresolution constellations where μ parameterizes the intracloud to intercloud ratio.

B. Multiresolution Modulation

Multiresolution modulation has been investigated in [1], [3], [4]. Some multiresolution constellations are shown in Fig. 3. Due to the different distances between the points in the signal space representation, the symbols exhibit different levels of noise immunity in the AWGN channel.

Our framework supports the overlaying of forward error correction codes (FEC) and can be very naturally combined with trellis coded modulation techniques [1], [3]. In this work, however, we restrict ourselves to achieving noise immunity solely through the idea of constellation clouds (see Fig. 3). We impose this intuitively appealing regular structure on the modulation constellation by characterizing it by the set of parameters $\{\mu_i\}$ that reflect the difference in noise immunity between consecutive layers. For the examples in Fig. 3, only one parameter μ is needed. In the interests of simplicity, in this paper, we consider the examples of 1-D (PAM) signal constellations since they have the same spectral efficiency as 2-D (quadrature amplitude modulation-QAM) signal constellations if a single side band (SSB) modulation technique is used. In order to construct a general 1-D signal constellation of size 2^L, we use $(L-1)$ μ-parameters, thus implying the possibility of $(L-1)$ "layers" with different noise immunity. An example of a multiresolution 8-PAM construction is shown in Fig. 4. Note that by taking the Cartesian product of 1-D constellations, we can construct multidimensional constellations in a straightforward way.

C. Source Modeling and Source Coding for Multiresolution JSCC

We have motivated the use of multiresolution source and channel coding in the introduction. For our problem, a multiresolution source coder can be characterized as producing

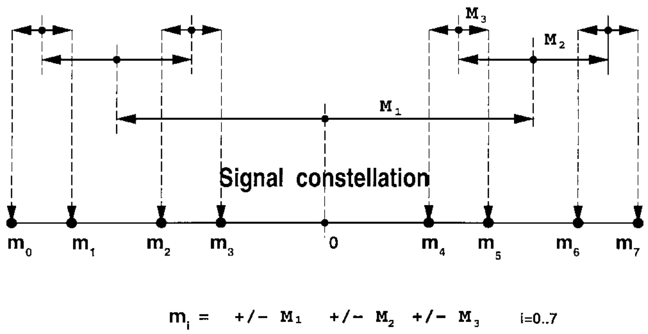


Fig. 4. Example of multiresolution constellation construction for 8-PAM. Three vectors M_i $i = 1, 2, 3$ are needed for the construction which results in two independent μ (μ_0 and μ_1)-parameters. We first start with 2-PAM defined by the length of M_1. After that, each point is split into two points to form 4-PAM with vector M_2. Finally, we split each point again to obtain 8-PAM with vector M_3, etc.

multiple information streams (of different degrees of "importance") associated with a single source of information. Note that this is consistent with existing popular layered video representation frameworks. Due to compatibility constraints, an multiresolution source coder will have some added redundancy in its output layers, which we desire to keep as low as possible. The multiresolution source coding problem in its different variations is a special case of the so-called multiple descriptions problem investigated in the information theory literature [17]–[19]. In the particular case of successive refinement [18], it was shown that rate-distortion optimality is achieved only for a special class of sources obeying a certain Markovian property.

An multiresolution representation can be achieved very naturally for the popular family of subband image coders. Different image subbands are fairly accurately modeled as i.i.d. sources with generalized Gaussian pdf with different shape parameters [20], [21]. The pdf for the zero-mean generalized Gaussian distribution with standard deviation σ and shape parameter ν is

$$f_X(x) = \left[\frac{\nu\eta(\nu, \sigma)}{2\Gamma(1/\nu)}\right] \exp(-[\eta(\nu, \sigma)|x|]^\nu) \tag{1}$$

where

$$\eta = \eta(\nu, \sigma) = \sigma^{-1}\left[\frac{\Gamma(3/\nu)}{\Gamma(1/\nu)}\right]^{1/2}$$

with

$$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}\,dt. \tag{2}$$

To ease computational complexity, it is also possible to model AC subbands as i.i.d. Laplacian sources ($\nu = 1$), whereas the DC subband is modeled as Gaussian i.i.d. source ($\nu = 2$) [21]. A popular approach to multiresolution representation of subband image data is based on bit planes, i.e., for a binary representation of subband coefficients, the most significant bit can be thought of as the base layer information, whereas the next bits are the refining information. This approach in the context of JSCC was considered in [22], where this representation was successfully used to unequally protect the source information. The allocation of channel codes

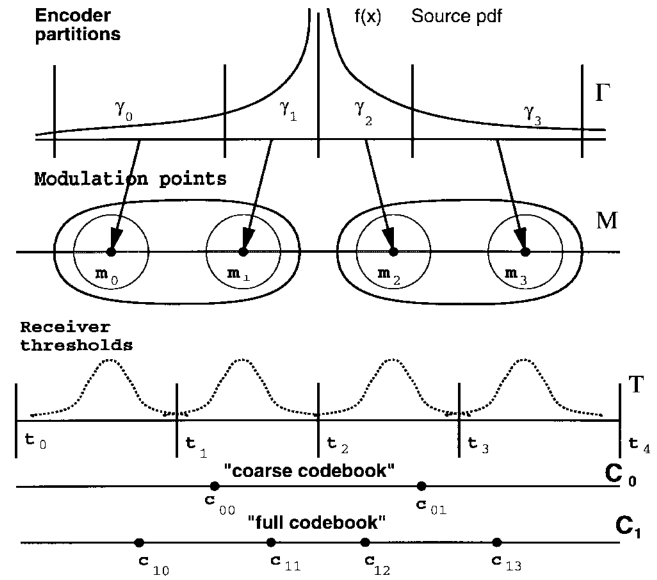


Fig. 5. Two resolution JSCC scheme for multiresolution 4-PAM and source codebooks having 2/4 levels.

there was done by estimating the "bit error sensitivities" of different layers. In this paper, we use a similar approach to multiresolution source coding, while combining it naturally with multiresolution channel coding through a multiresolution modulation.

D. Proposed Framework

Our framework is most easily explained through an illustrative example of a two-state AWGN channel model. This channel model is insightful as it can be easily generalized to the desired time-varying channel case by considering an N-state AWGN channel, as N gets sufficiently large.

1) Example of Four-Level Quantizer, 4 PAM, and Two-State AWGN Channel with Uninformed Transmitter: Here, we consider the source-channel coder illustrated in Fig. 5 and a two-state AWGN as in Fig. 1. Suppose the channel can be in one of only two different states. In each state s (with p_s denoting the probability of occurrence of the state s), the channel is AWGN with a different noise variance σ_s^2, with the states being labeled "good" and "bad." Suppose that the receiver knows the actual channel state, whereas the transmitter has knowledge of only long-term channel statistics, i.e., the state probabilities. Suppose we want to transmit an i.i.d. (scalar) source X with pdf $f(x)$ quantized to four levels through this channel using a 4-PAM modulation constellation, assuming an optimal one-to-one mapping between the encoder-partitioning $\{\gamma_i\}_{i=0}^3$ and the constellation points $\{m_i\}_{i=0}^3$, as shown in Fig. 5. The joint encoder/modulator operation is, therefore, to partition the source x into intervals $\{\gamma_i\}_{i=0}^3$ and map each γ_i to the corresponding constellation point m_i. Since both the channel and the source are memoryless, we use an optimal hard-decision demodulator that declares, based on optimized demodulator thresholds $\{t_i\}$, which m_i was transmitted. In general, the receiver thresholds have to be chosen differently for different channel states since they depend on the distribution of the received signal and, hence, on the channel noise power. (At low CNR, for instance, even distant points start to contribute significantly to the distribution of the received signal amplitude. On the other hand, for high CNR, only nearest neighbors have to be taken into account.) Finally, the decoder performs a one-to-one mapping between the m_i's and the source reconstruction codewords c_i. Let us, for now, assume that both codebooks have the same number of reconstruction points c_i. We will address the multiresolution scenario shortly. Given this framework, the question is how to design the end-to-end system in order to minimize the overall distortion (due to source quantization *and* channel noise) for a fixed average transmission energy (per channel symbol). The system parameters include the following:

1) the source encoder partitions $\Gamma = \{\gamma_i\}$;
2) the channel modulation constellation $\mathcal{M} = \{m_i\}$;
3) the receiver decision thresholds $\mathcal{T}_s = \{t_{s,i}\}\ \ s = 0, 1$;
4) the source decoder codebooks $\mathcal{C}_s\ \ s = 0, 1$ containing the reconstruction codewords $\{c_{s,i}\}$.

Our goal is to minimize the expected distortion between source sample X and its replica $\hat{X}$ (we denote expectation $\mathcal{E}\{\}$)

$$\min_{\mathcal{M}, \Gamma, \mathcal{T}_s, \mathcal{C}_s} \left[\mathcal{E}\{D(X, \hat{X})\} = \sum_{s=0}^{1} p_s \sum_{i=0}^{3} \int_{\gamma_i} \cdot f(x) \sum_{j=0}^{3} (x - c_{s,j})^2 \, \mathrm{Prob}_s(j|i) dx \right] \tag{3}$$

where the transitional probabilities of decoding $c_{s,j}$ given that $x \in \gamma_i$ in the channel state s are in the form

$$\mathrm{Prob}_s(j|i) = \mathcal{Q}[(t_{s,j} - m_i)/\sigma_s] - \mathcal{Q}[(t_{s,j+1} - m_i)/\sigma_s] \quad \text{where } \mathcal{Q}(x) = \mathrm{erfc}(x/\sqrt{2})/2 \tag{4}$$

subject to the fixed energy constraint

$$E_{av} = \sum_i m_i^2 \int_{\gamma_i} f(x)\, dx. \tag{5}$$

To solve the constrained problem of (3) and (5), we introduce the Lagrange multiplier λ [23] and solve the unconstrained problem of the form

$$\min_{\mathcal{M}, \Gamma, \mathcal{T}_s, \mathcal{C}_s} [J(\mathcal{M}, \Gamma, \mathcal{T}_s, \mathcal{C}_s)] = \min_{\mathcal{M}, \Gamma, \mathcal{T}_s, \mathcal{C}_s} [\mathcal{E}\{D(X, \hat{X})\} + \lambda E_{av}] \tag{6}$$

where parameter $\lambda \geq 0$ is chosen to satisfy the energy constraint (5) and can be interpreted as a coefficient that trades off energy for distortion in the optimization process. If a solution for the unconstrained problem (6) exists, it is also a solution to the constrained optimization problem in (3) and (5) (see [23] for details in the context of compression). Given the expression for the cost function (6), the following encoding and decoding rules can be derived (details are given in the Appendix)

ENCODING:

Assign x to γ_i such that $i =$

$$\arg\min_l [\lambda (m_l)^2 + \sum_{s=0}^{1} p_s \sum_{j=0}^{3} \mathrm{Prob}_s(j|l)(x - c_{s,j})^2] \tag{7}$$

i.e., where i is a value of l for which the expression in square brackets is minimized.

DECODING:

$$c_{s,j} = \frac{\sum_{i=0}^{3} \mathrm{Prob}_s(j|i) \int_{\gamma_i} x f(x)\, dx}{\sum_{i=0}^{3} \mathrm{Prob}_s(j|i) \int_{\gamma_i} f(x)\, dx}$$

$$\text{i.e., } c_{s,j} = \mathcal{E}\{X|\text{ received point is } j\} \qquad s = 0, 1. \tag{8}$$

Note that (7) does not give the explicit boundaries for regions γ_i's. The explicit form can be derived by setting the partial derivatives of J with respect to boundary points to zero. However, this is only a necessary condition for the minimum, whereas the sufficient condition is not always satisfied. Unfortunately, a closed-form solution to this problem is not tractable so that numerical methods have to be used. It is insightful to note that the "price" for assigning x to γ_i is a weighted sum of distortion and energy associated with this decision, which is a variation of the entropy-constrained vector quantization (ECVQ) [24] problem, where the equivalent "entropy" term in ECVQ is replaced by an "energy" term. The decoding rule (8) is a variation of the weighted centroid condition in the channel-optimized Lloyd–Max quantizer.[3] It is possible to formulate the optimal encoding (values of m_i) and decoding ($\mathcal{T}_s$) rules given the cost function in (6), but it appears that an analytical solution is not feasible because of nonlinear relations (through the Q-function) between transitional probabilities $\mathrm{Prob}_s(j|i)$ in the optimal receiver and channel coder parameters. On the other hand, if a suboptimal receiver (for example, a maximum likelihood receiver) is used, then the performance of the system degrades dramatically because constellation points are not equiprobable.[4] Instead, in the following, we propose an iterative algorithm (which is similar to Lloyd's algorithm) to solve the problems of the type in (6).

For our simple example, there are only two different states, each represented by an equivalent AWGN channel. Clearly, the best one can do is to design two *different* optimal decoders for each channel state jointly with a *single* encoder (since the transmitter cannot access the CSI). The receiver should then use the perfect CSI to switch between these two optimal decoders. Such a design, while manageable for a two-state AWGN channel model, is clearly impractical when the number of states gets large (as needed to approximate the desired channel arbitrary closely) since this would require a separate design for each channel state and would increase the complexity. An obvious way of alleviating this problem is to allow only a few different codebooks and devise an optimal decoding strategy for deciding the codebook that should be used in each channel state. However, instead of designing several full resolution codebooks (i.e., having maximum number of reconstruction levels), we propose to design a hierarchy of multiresolution codebooks and allow the receiver to choose among them using the instantaneous CSI. The motivation for this comes

[3] Our approach to obtain the necessary conditions for source coder optimality is similar to previous results in [8] and [9].

[4] The probability of symbols typically decreases as its magnitude increases because of typical source pdf shapes and the energy constraint.

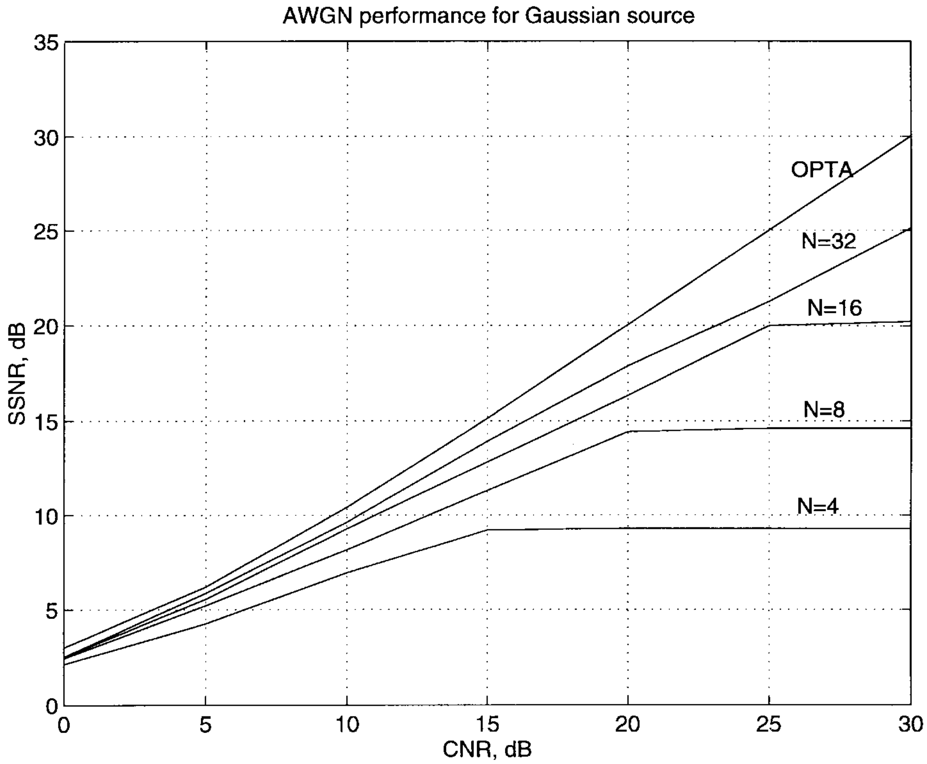


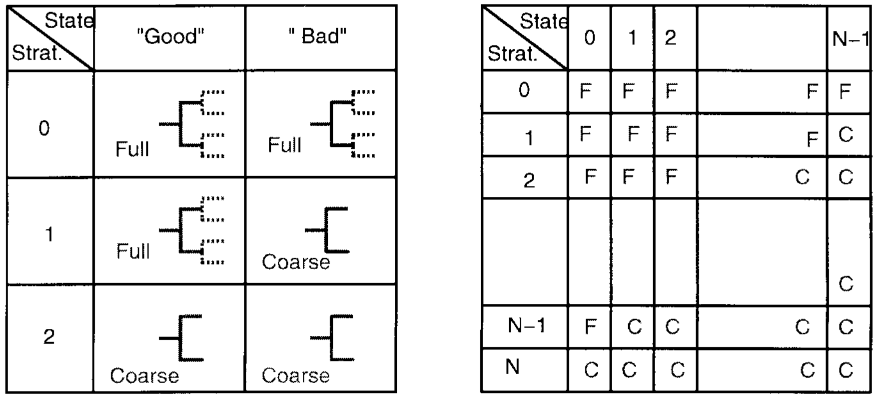
Fig. 6. Comparison of the OPTA and actual performance for 4-, 8-, 16-, and 32-PAM systems jointly optimized for an AWGN channel and a Gaussian source. Note that at lower CNR's, there is not much gain in increasing the number of levels in the coder. Another observation is that there is relatively small loss in performance of PAM systems compared with the OPTA bound in a wide range of CNR's (e.g., to 15 dB in CNR). At high CNR's, the loss is dominated by the quantization noise rather than the channel noise.

from an attempt to match the source and channel coder in a natural and efficient way by preserving the intuitive structure of "clouds" and "satellites" in both domains. A justification of our proposed framework is that we lose little performance by reducing the number of reconstruction levels in the "low resolution" codebooks because the "full resolution" design typically degenerates to a "low resolution" solution as the channel degrades sufficiently. The degradation in performance is not significant, even compared with the optimized informed transmitter case (which obviously upper bounds the performance of any design for the uninformed transmitter case), as shown in our simulations in Section V. Further, the proposed multiresolution framework is highly desirable for wireless multicast and broadcast applications, although this paper deals with the point-to-point wireless channel.

The intuitive explanation for using a multiresolution structure may be stated as follows. Suppose the channel is very noisy. If we decrease the number of possible decisions at the receiver, two things happen: The fidelity of the source representation is decreased, which results in increased distortion, but at the same time, the signal immunity to the channel noise is increased because of increased distance between points in the signal constellation. Thus, there could be situations where the fidelity of the source representation during the "good" channel state is sacrificed to improve the performance in the "bad" state by a somewhat larger amount of the cost-function J. In those cases, it pays to have an embedded multiresolution design. We would like to point out that similar observations have been made in [9] and [11] for BSC with fixed transitional probabilities and in [14] for the AWGN channel in a single-resolution system, where it was shown that the optimal encoder may reduce the alphabet size to increase the error resilience if the channel is very noisy. In our framework, the tradeoff between "good" and "bad" channel state performance is done using a multiresolution framework through embedding in the modulation space.

To further validate our approach, we compare the optimal performance theoretically attainable (OPTA) and the performance of the optimal JSCC PAM systems considered in this work (which we describe later) for the special case of a Gaussian source and a Gaussian channel (see Fig. 6). Observe that we do not lose too much by using coarser resolution coders in the lower CNR regime, where the performance is dominated by the channel noise. Note further that the overall inefficiency due to quantization and use of a finite constellation is not significant for a wide range of CNR's of interest (e.g., for $N = 32$, it is less than 1.6 dB for CNR's up to 15 dB).

To summarize, in our example, a two-resolution choice offers a tradeoff between source coder fidelity and channel noise immunity. The choice of the full-resolution (four-level) codebook $\mathcal{C}_0$ and its associated full four-point constellation is biased toward increased source (and decreased channel) resolution, whereas the opposite is true for the coarse-resolution (two-level) codebook choice $\mathcal{C}_1$ and its associated two-"cloud" constellation representation. An attractive solution is to have the receiver devise a (*predesigned*) rule that assigns the optimal codebook choice to each of the two channel states $s = 0, 1$ (and, in general, for each of N states $s = 0, \cdots N-1$) from



(a) (b)

Fig. 7. (a) Strategy diagram for the illustrative example of two-resolution case and two-state channel. (b) Possible strategies for N-state channel and two-resolution case. Note the special structure of the possible strategies pattern that allows simplification of the search for the optimal strategy.

among $\mathcal{C}_0$ and $\mathcal{C}_1$ (and, in general, $\mathcal{C}_0, \mathcal{C}_1, \cdots, \mathcal{C}_{L-1}$, where L is the number of resolutions). It is possible to think of the polarity of the source symbol ($+/-$) as being the "coarse" source information and the quantized magnitude as the refinement information, i.e., this is the case when a multiresolution representation is similar to successive approximation. Given that the coarse information is received correctly, we can refine it further if the channel is reliable enough.

Before proceeding to the more general case of the N-state AWGN channel, we introduce the notion of *strategy*, which is basically a rule that assigns a particular source/channel coder resolution to a given channel state. The number of different allowed strategies explores the tradeoffs between the complexity of the system and the distortion between the original and decoded data. Suppose, for instance, that in our example of Fig. 5, we allow only two different reconstruction codebooks of four and two levels, respectively, with corresponding channel decoders. Then, the possible strategies are the following.

1) $\mathbf{S}_0 = (\mathcal{C}_0, \mathcal{C}_0)$, i.e., a full-resolution codebook is used in "good" and "bad" states (i.e., "bad" state is still not "bad enough").
2) $\mathbf{S}_1 = (\mathcal{C}_0, \mathcal{C}_1)$, i.e., a full-resolution codebook is used in "good" state and coarse resolution in "bad" state.
3) $\mathbf{S}_2 = (\mathcal{C}_1, \mathcal{C}_1)$, i.e., coarse resolution codebook is used in "good" and "bad" states (which is a subcase of S_0, but we keep it for generality).

These possibilities are illustrated in Fig. 7(a). The extensions to multistate case Fig. 7(b) and more than two resolutions (not shown) are straightforward.

We need to try all possible strategies and pick the best one. For a particular channel and source model, the calculations can be performed *off line* and then stored so that only a lookup table is needed in real-time operation. The previous case is easily extended to the N-channel case, where imposing the "monotonicity" requirement that the optimal strategy at a better (i.e., less noisy) channel state would necessarily use the same or better resolution codebook choice, can result in dramatically reduced search complexity for the optimal $\mathbf{S}$. Note that by applying additional constraints for the lower resolutions solution to be a subset of the higher resolution one, we can obtain an "embedded" design having a successive refinement property, i.e., when it is enough to specify a single codebook.

III. Uninformed Transmitter Case for a Single Source

A. Problem Statement for Multistate AWGN Channel

Let, as before, $\mathcal{M} = \{m_i\}$ refer to the multiresolution constellation set with constellation points $\{m_i\}$ characterized by parameters $\{\mu_i\}$ (see Figs. 3 and 4). Let $\Gamma = \{\gamma_i\}$ refer to the encoder partitions, where γ_i is assumed to be mapped one-to-one to constellation point m_i. Since perfect CSI is available at the receiver, we can optimize the choice of demodulator thresholds t_i as well as the (predesigned) multistate "resolution" strategy $\mathbf{S}$.

As a straightforward extension of the two-resolution, two-state AWGN example, the strategy $\mathbf{S}$ for an L-resolution, N-state AWGN case can be written as an N-vector whose entries are selected from among $\mathcal{C}_0, \mathcal{C}_1, \cdots, \mathcal{C}_{L-1}$ (we omit the explicit optimization of demodulator thresholds hereafter assuming that each $\mathcal{C}_i$ corresponds to $\mathcal{T}_i$), where each entry corresponds to the particular resolution picked for the particular channel state. The problem is to find, for a target average transmission energy E_{av}, the optimal choices for $\mathcal{M}$, Γ, and $\mathbf{S}$ such that the average end-to-end expected distortion $D(X, \hat{X})$, which is a weighted average of the conditional distortions given the channel states, between the input X (which is assumed to be a scalar random variable having a pdf $f(x)$) and its estimated approximation $\hat{X}$, is minimized

$$\min_{\mathcal{M}, \Gamma, \mathbf{S}} \left[\mathcal{E}\{D(X, \hat{X})\} = \sum_s p_s\, \mathcal{E}\{D(X, \hat{X}|s, \mathbf{S})\} \right] \tag{9}$$

subject to the fixed energy constraint

$$E_{av} = \sum_i m_i^2 \int_{\gamma_i} f(x)\, dx \tag{10}$$

where, again, p_s is the probability of the channel being in the sth state.

B. Problem Solution

We convert the constrained problem of (9) into an unconstrained one using the Lagrange multiplier method using the intermediate variable $\lambda \geq 0$ [23], which trades off distortion for energy. That is, the equivalent unconstrained problem is to minimize $J = D + \lambda E$. The solution for a fixed value of the Lagrange multiplier λ and fixed values of constellation parameters $\{\mu_i\}$ can be found by an iterative design procedure similar to the Generalized Lloyd algorithm (GLA) [7] and is described in the algorithm below. For each value of λ, we perform a gradient-based search on $\{\mu_i\}$ until we converge to the locally optimal design for some target signal energy E_{final}. If E_{final} is not the desired energy E_{av}, than our guess of parameter λ is not matched to E_{av} so that we need to change λ appropriately (and perform iterations again) to meet the desired energy E_{av} constraint. Often, we will want to obtain the complete "energy-distortion" (E-D) curve for a particular system in which case we just sweep λ over a finite grid of values in the range of interest. This is the direct dual of rate-distortion curves used for source coding [23], [25]. The above optimization procedure is repeated for different strategies $\mathbf{S}$ to find the best strategy. All calculations are performed *off line* with results (optimized design "codebooks" and receiver thresholds) being stored in the form of lookup tables. In a real-time operation, only table lookup operations and trivial operations like scaling are needed.

We would like to comment on the relationship of our proposed algorithm to other quantizer design algorithms for discrete alphabet time-varying channels proposed in the literature [10], [11]. Our philosophy is similar to those in [10] and [11] for any fixed choice of the modem parameters. However, we additionally explore the space of all admissible choices for the modem parameters within the constraints of our imposed regular structure and the given transmission power budget and find the best solution. Furthermore, this is done not separately but jointly with the source codebook design.

C. JSCC Algorithm

Step 0) Initialize strategy $\mathbf{S}$ defining which of the L multiresolution codebooks (from among $\{C_i\}_{i=1}^{L}$) is assigned to each of the N channel states $\{s_i\}_{i=1}^{N}$.

Step 1) Initialize the value of the Lagrange multiplier λ.

Step 2) Initialize the constellation parameters $\{\mu_i\}$.

Step 3) Initialize the encoder partitions Γ, decoder reconstruction codebooks $\mathcal{C}_r$ for each resolution r, receiver thresholds $\mathcal{T}_s$ for each channel state s, and modulation constellation $\mathcal{M}$.

Step 4) For a fixed $\mathcal{M}$, apply a MR-JSCC-modified version of the Lloyd–Max algorithm to iteratively optimize the encoder partitions γ_i and the decoder reconstruction levels $\mathcal{C}_r$ together with optimization of demodulator thresholds $\mathcal{T}_s$ for each channel state s so that the cost function is minimized. The distortion can be written as

$$\mathcal{E}\{D\} = \sum_r \sum_{s \in S_r} p_s \left[\sum_i \int_{\gamma_i} f(x) \sum_j (x - c_{r,j})^2 \cdot \text{Prob}_s(j|i) dx \right]$$

where

r resolution index;

S_r subset of states for which the codebook $\mathcal{C}_r$ with codewords $c_{r,j}$ is used (this is determined by the strategy $\mathbf{S}$);

$P_s(j|i)$ *a priori* probability of receiving m_j given that m_i is sent.

The expression in brackets is the distortion in a particular state s, and we average it over all possible resolutions r and channel states $s \in S_r$ mapping to these resolutions. Note that the transitional probabilities $P_s(j|i)$ depend on the average channel CNR_{av} in state s (this is a function of the assumed multistate channel model) as well as on the receiver decision thresholds $\mathcal{T}_s$.

We use the following two-step iterative procedure for the source coder optimization (derivations are similar to the ones in the Appendix):

a) Optimize the reconstruction codebooks $\mathcal{C}_r$ for a fixed encoder partitioning Γ using a weighted centroid condition (WCC). Then, for each codeword at resolution r, we can write

$$c_{r,j} = \sum_{s \in S_r} p_s \left[\sum_i \alpha(i|j) \beta_i \right] \tag{11}$$

where β_i are the (noiseless channel) centroids, and $\alpha(i|j)$ are the *a posteriori* probabilities of $x \in \gamma_i$ given that $c_{r,j}$ has been received in rth state.

b) Assign source input x to the nearest encoder partition γ_i in the Lagrangian sense

$$i = \arg\min_l \left\{ \lambda (m_l)^2 + \sum_r \sum_{s \in S_r} p_s \sum_j \cdot [\text{Prob}_s(j|l)(x - c_{r,j})^2] \right\}. \tag{12}$$

A separate optimization of $\mathcal{T}_s$ is performed implicitly in the calculation of the transitional probabilities $\text{Prob}_s(j|i)$. At this step, the value of each decision threshold is found that minimizes the value of the cost function J, given that all other parameters are fixed.

Step 5) Do the iterations of Step 4 until local convergence.

Step 6) In Steps 3–5, perform a gradient search over the constellation parameters $\{\mu_i\}$, and pick values $\{\mu_i^*\}$ for which the cost function is minimum.

Step 7) Do a convex search over $\lambda \geq 0$ (Steps 2–7) to find the optimal λ^* that satisfies E_{av}.

Step 8) Search all candidate strategies, and pick the best (repeat Steps 1–7).

Note that the search for the best strategy (Step 8) does not have to be exhaustive but is instead a fast convex search due to the monotonic relationship between states and resolutions (see Fig. 7). Specifically, this means that a state with higher CNR cannot be assigned coarser resolution than a state with lower CNR, and hence, finding the best strategy is equivalent to finding the breakpoints for the intervals corresponding to different resolutions (which appear in descending order, starting from the best channel state). For example, for the two-resolution case such as 4/2 PAM in Section II-D1, only one such break point has to be found, for which a fast line searching technique can be used.

D. Simulation Results for Uninformed Transmitter Case

We present simulation results for a multistate AWGN channel that approximates a flat fading Rayleigh channel model with average CNR denoted by CNR_{av}. We assume a unit variance Laplacian source quantized to two resolutions using 8/4-level reconstruction codebooks $\mathcal{C}_{\mathrm{full}}$/ $\mathcal{C}_{\mathrm{coarse}}$ and an 8-PAM constellation. We approximate the Rayleigh channel by a 20-state AWGN model, which we found experimentally to be a sufficiently good approximation. Specifically, to validate our approximation, we simulated PAM transmission using our multistate AWGN approximation and compared it with the theoretical BER for the Rayleigh channel. We found that the approximation error in BER is at most 10% in the typically interesting CNR range of 3–15 dB.

Fig. 8 shows the results of applying the MR-JSCC algorithm described in Section III-C. Two reference systems are presented for comparison. The separately optimized reference system in Fig. 8(a) and (b) has uniform PAM constellation and source optimal quantizer. The single resolution reference system in Fig. 8(a) and (b) was designed using the JSCC algorithm III-C with a single reconstruction codebook (high resolution) and a uniform PAM constellation (but with optimized receiver decision regions selected using CSI). The MR-JSCC scheme is based on the best strategy for each CNR_{av}. Note how significant gains can be obtained for a wide range of CNR_{av}'s. The gains of the designed system are the result of using multiresolution codebooks and optimization of the signal constellation. The optimal MR-JSCC system does not have uniform modulation points, especially at low CNR_{av}'s. Comparing the two reference systems, notice how important the channel optimized design of the source coder is, especially in the low CNR region, where the separately designed system breaks down. It is interesting to note that the performance of the separately designed reference system in low CNR_{av} regions degrades if the number of quantization levels/modulation points is increased. This happens because the channel noise contribution to the overall distortion in low CNR_{av} regions is much bigger than the quantization noise. JSCC systems do not expose this "artifact" thanks to the joint source/channel coder design.

We would like to note that for the special case of a Gaussian memoryless source and an AWGN channel, the mapping from the source domain to the signal constellation will tend to be linear as the number of levels in source/channel coder increases. Actually, linear one-to-one mapping (when the channel signal amplitude is determined by scaling the source sample amplitude) will achieve the optimal performance theoretically attainable for this source and AWGN channel with energy constraint [12]. Unfortunately, for other source distributions, linear mappings are no longer optimal, and optimal mappings are not known in most cases but can be determined numerically using Algorithm III-C.

Fig. 9 shows the relative frequency of usage of the full and coarse codebooks in the optimal design. As expected, the coarse codebook is used more frequently for Rayleigh channels having lower CNR_{av} values. As the channel condition improves, the $\mathcal{C}_{\mathrm{coarse}}$ codebook is used more sparingly. Note that for every particular multistate channel characterized by average CNR_{av}, there is an optimal strategy that minimizes the expected distortion. For example, if the Rayleigh channel has $\mathrm{CNR}_{av} = 10$ dB, the optimal strategy (as shown in Fig. 9) is to use the coarse resolution codebook $\mathcal{C}_{\mathrm{coarse}}$ 10% of time (i.e., during the worst two states in the 20-state model). Fig. 10 shows the regions of optimal strategies corresponding to different usages of coarse/full resolution codebooks, highlighting the locations of critical points where resolution transitions occur.

IV. Uninformed Transmitter Case for a Composite Source

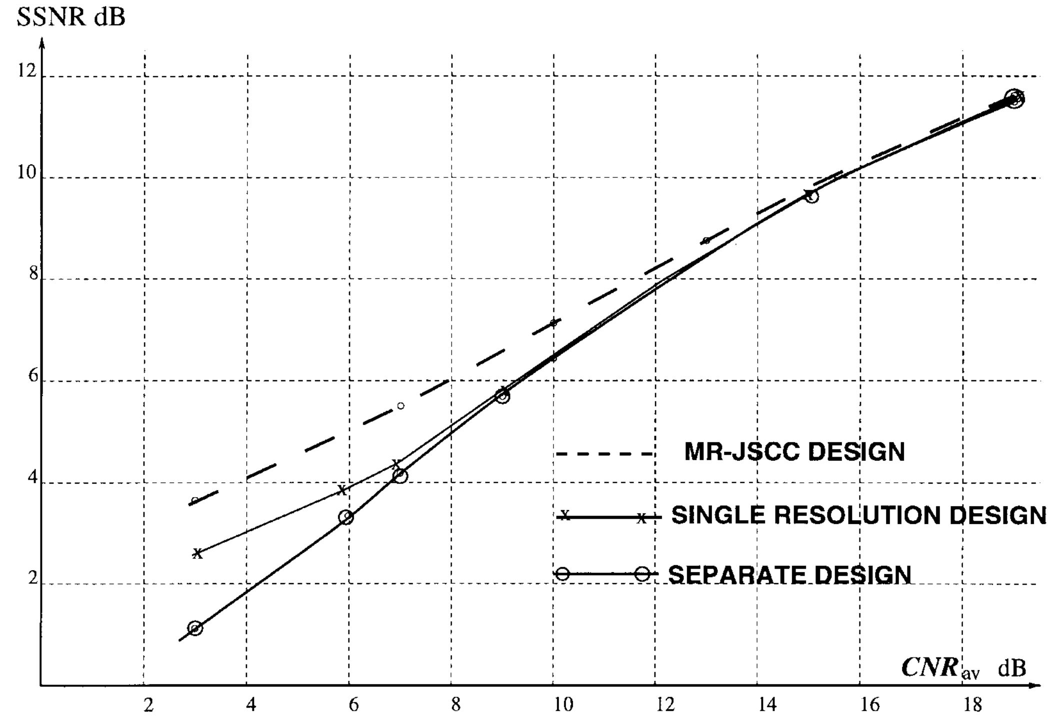
Consider the following modification of the uninformed transmitter problem from the previous section. Suppose that instead of a single scalar source X, we wish to transmit a composite source consisting of several i.i.d. sources X_k from the same pdf but having different parameters. A situation like this is realistic for image transmission when, for example, a subband decomposition is used and the coefficients of several subbands to be sent can be modeled with the same family of pdfs, e.g., generalized Gaussian (GG) [see (1)] but having different parameters. Since the source information is available at the transmitter, for an optimal design, it should be used in the power control algorithm. We can formulate the following problem

$$\min_{\mathcal{M}_k,\,\Gamma_k,\,\mathcal{T}_{k,s},\,\mathcal{C}_{k,s}} \Big[J = \sum_k \mathcal{E}\{D_k\} + \lambda \sum_k \mathcal{E}\{E_k\}\Big] \qquad (13)$$

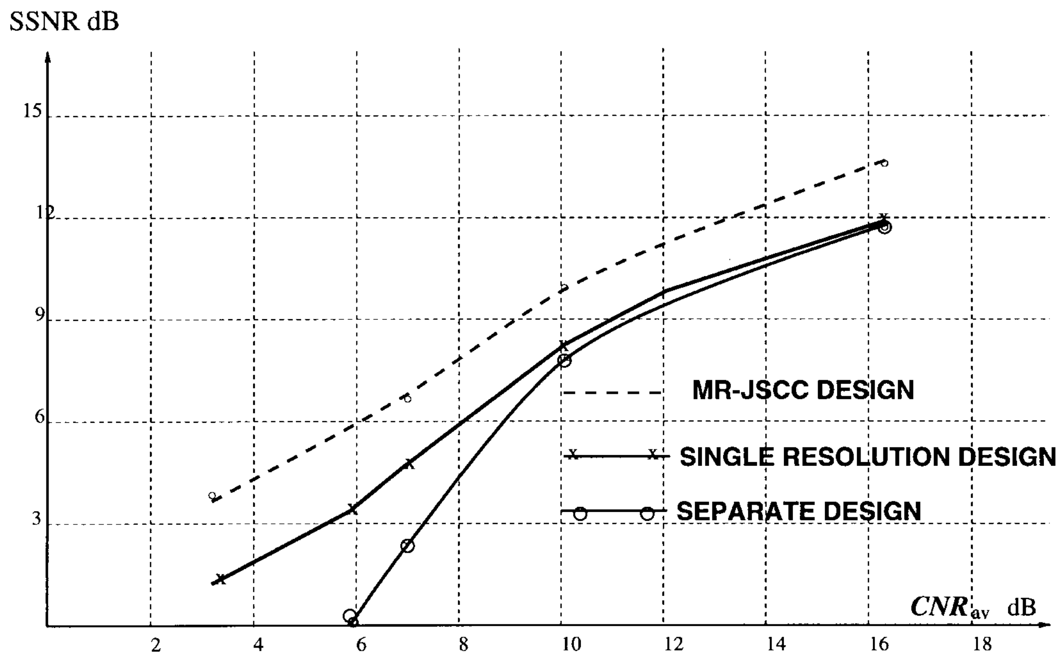
where $\mathcal{E}\{E_k\}$ is the expected energy spent on transmission of the coding unit of source X_k.

Since the cost function is additive, we can easily extend the results obtained for the single source optimization case of Section III. The JSCC algorithm, which has been described in Section III-C, is used to obtain the E-D curve for a single i.i.d. source. A typical plot is presented in Fig. 11, where each point corresponds to a particular (optimized) value of λ (and resulting $\mathcal{M}$, Γ, $\mathbf{S}$, $\mathcal{C}_r$ and $\mathcal{T}_r$). If only the variance of the source pdf is changed, parameters $\mathcal{M}$,$\mathbf{S}$, and $\mathcal{T}_r$ remain the same, whereas the source coder parameters Γ and $\mathcal{C}_r$ are scaled by the square root of the source variance relative to those corresponding to the unit variance case. The E-D curve is scaled in the direction of the distortion axis. This observation allows us to use a single precomputed E-D curve to generate E-D curves for all components X_k.

The problem can now be formulated as finding the points on E-D curves for each source X_k such that the total energy spent is less then or equal to the energy budget, and the expected distortion $\mathcal{E}\{D\}$ is minimized. The optimal allocation is the well-known "constant slope" solution [23], [25], where the individual operating points "live" at constant slope on the individual E-D curves. This approach has been successfully



(a)



(b)

Fig. 8. Performances of designed and reference systems for the case of a Rayleigh channel with average CNR given by CNR_{av}, unit variance Laplacian source: (a) PAM-8 and 8/4-level $\mathcal{C}_{\mathrm{full}}/\mathcal{C}_{\mathrm{coarse}}$ and (b) PAM-32 and 32/16/8/4/2-level codebooks $\mathcal{C}_{i=0}^{4}$. The separately optimized reference system consists of a single resolution, source optimized quantizer, and a uniform PAM constellation with optimized receiver thresholds. The single resolution reference system consists of a channel optimized quantizer and a uniform PAM constellation with optimized receiver thresholds. The MR-JSCC scheme is based on the best strategy for each CNR_{av}. Note that at low CNR_{av}, the separately optimized reference system with fewer levels performs better than the reference system with more levels because it "absorbs" less channel noise.

used in solving problems of optimal resource allocation with additive cost function, for example, in rate allocation for the minimum distortion [23], [25]. The process is illustrated in Fig. 12.

To illustrate the difference in performance for a composite memoryless source, we have simulated the transmission of the AC subbands of the wavelet transform of the "Lena" image. We model subbands as i.i.d. Laplacian sources and perform optimal energy allocation among composite source components using the previously described Lagrangian techniques for E-D curves. The Laplacian model is often used for modeling the AC subband coefficients for quantization and entropy coding [21].

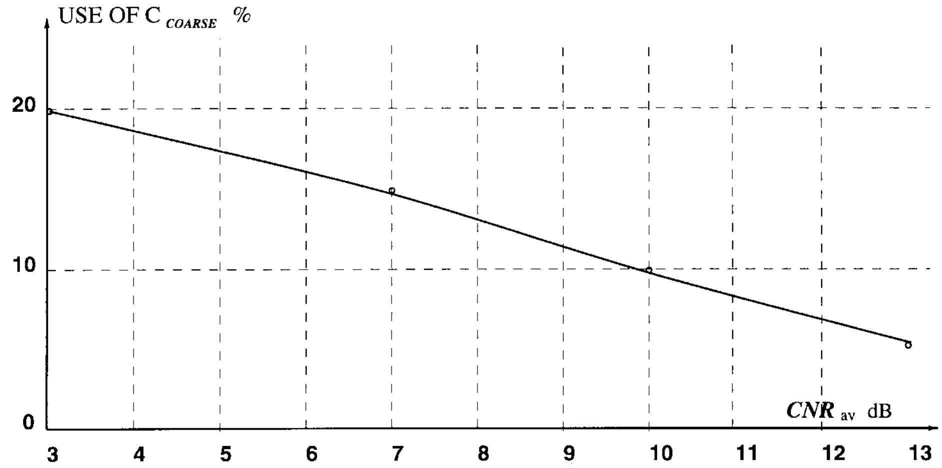


Fig. 9. Frequency of usage of coarse reconstruction codebook $\mathcal{C}_1 = \mathcal{C}_{\text{coarse}}$ in the optimal strategy for unit variance Laplacian source and 8-PAM for Rayleigh channel. $\mathcal{C}_0 = \{c_i\}_{i=0}^{7}$, $\mathcal{C}_1 = \{c_i\}_{i=0}^{3}$.

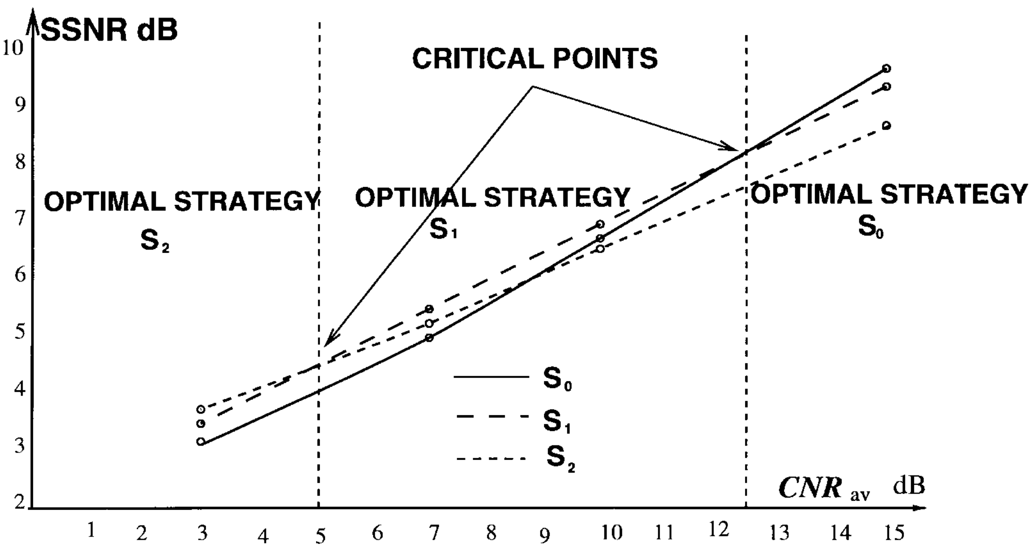


Fig. 10. Regions of optimal strategies bounded by critical CNR_{av} values $\mathbf{S}_0$, $\mathbf{S}_1$, $\mathbf{S}_2$ use full resolution codebook $\mathcal{C}_0 = \{c_i\}_{i=0}^{7}$ for 100, 90, and 80% of time, respectively. Note the positions of "critical" CNR_{av} points where resolution transitions occur.

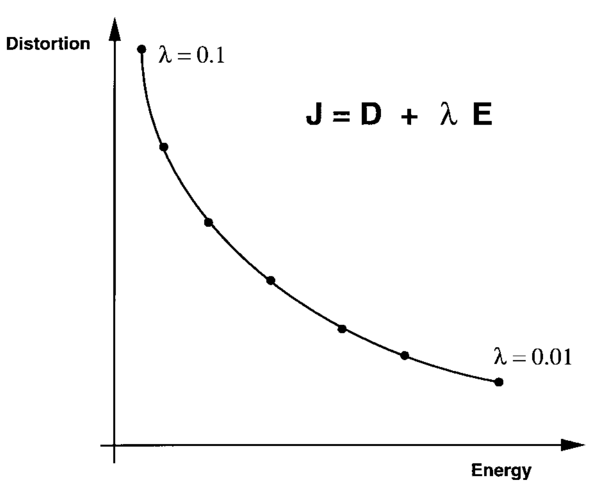


Fig. 11. Typical energy/distortion curve. Each point corresponds to a particular value of λ in the cost function.

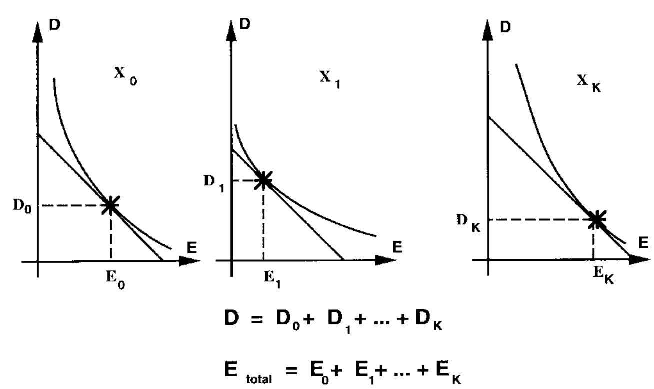


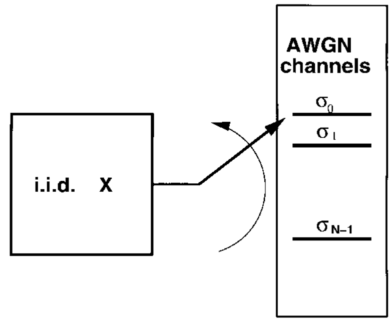
Fig. 12. Optimal energy allocation for multiple sources. At optimality, operating points "live" at a constant slope on their respective E-D curves. Total energy E_{total} is adjusted by choosing the appropriate parameter λ.

The computer-simulated results of the joint source-channel coded optimization are illustrated in Fig. 13(a) and (b) for the transmission of a wavelet image decomposition in a multistate AWGN approximation of the Rayleigh channel. The MR-JSCC optimized system [Fig. 13(b)] uses several decoding resolutions, depending on the channel condition, and thus, it

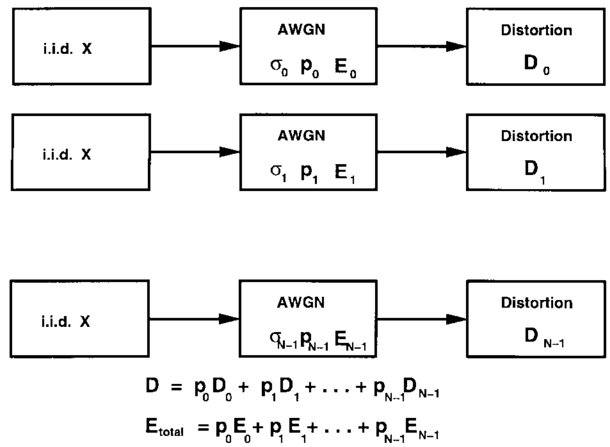
(a)

(b)

Fig. 13. Transmission through Rayleigh channel: (a) Single resolution system, 28.3 dB PSNR. (b) Multiresolution jointly optimized system, 30.5 dB PSNR. SR system uses 5-bit quantizer with 32-PAM. multiresolution system uses five resolutions of reconstruction levels in the quantizer with 5-bit being the highest resolution with 32-MRPAM. Both systems operate with the same average power.



(a)



(b)

Fig. 14. Illustration for the problem with single source and informed transmitter. The system (a) is equivalent to the system (b). The i.i.d. source x is sent through N separate channels with known noise variance σ_i and probability of occurrence p_i. The optimization task is to distribute the total energy E per coded unit so that the expected distortion D is minimized.

avoids catastrophically large errors during deep fades. The single-resolution system [Fig. 13(a)] tries to use the highest resolution even in really bad channel conditions ("dots" on the plot). In the simulations, we have assumed that the DC subband is sent "losslessly" and that perfect channel state information is available. This result clearly shows the *relative* gain of the multiresolution system over the single resolution for the time-varying channels.

V. Informed Transmitter Case

Suppose that a feedback channel is available, and the transmitter is aware of the instantaneous channel state (we address slow-fading channels in this model) and that the problem is again to minimize the end-to-end distortion while satisfying the average energy per coding unit constraint in (9) and (10). Let us discuss the single i.i.d. source case first. The original problem shown in Fig. 14(a) can be formulated equivalently in the following way: Given N AWGN channels with noise variances σ_i, which are used to transmit an identical source during the fractions of time determined by the probabilities of a particular state p_i (which is obtained from the multistate AWGN model), allocate the total energy E such that the total distortion D is minimized. We illustrate this equivalent scheme in Fig. 14(b). By applying the algorithm in Section III-C for an AWGN channel, we obtain the E-D curve of a single i.i.d. source being transmitted over the single state AWGN channel. To obtain the E-D curves for other channels, we simply scale the energy axis (for same family of pdf's) by the appropriate factor as described in Section IV. Then, it is possible to perform an optimal unequal energy allocation between different channels in Fig. 14(b) similar to that illustrated in Fig. 12. As before, at optimality, in all channel states, the slope of the E-D curves should be the same.

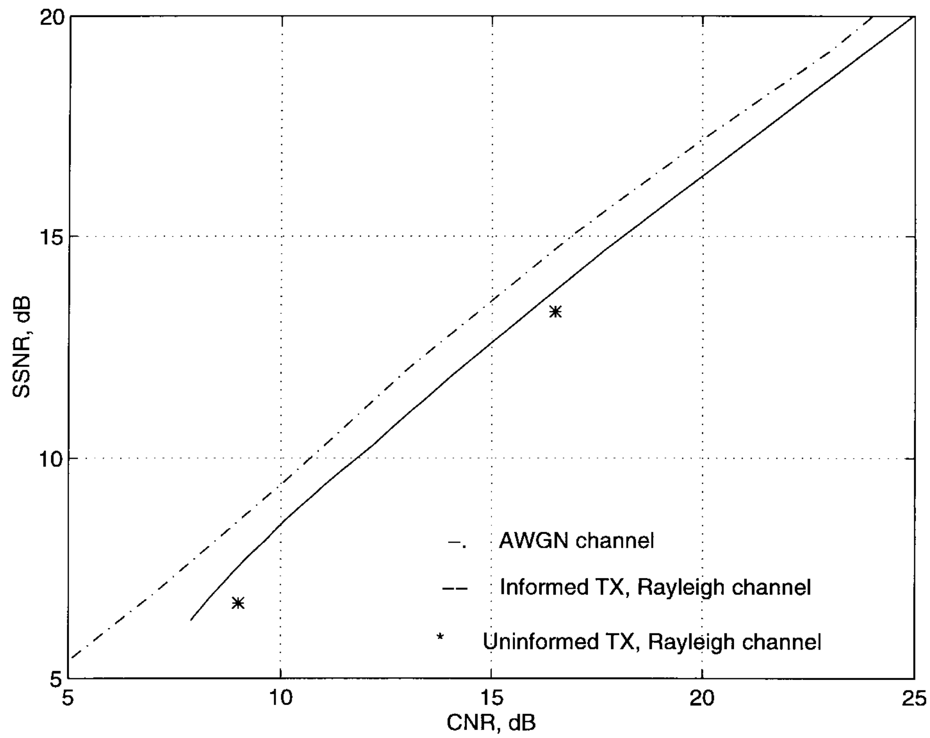


Fig. 15. Performance of 32-PAM system for Laplacian source. The power loss due to Rayleigh fading is about 2 dB for informed transmitter case and 2.5 dB for uninformed transmitter relative to an AWGN channel performance. The CSI availability does not significantly increase the performance of the jointly designed system.

The results of simulated performance for the single i.i.d. Laplacian source and informed transmitter are presented in Fig. 15. We have used 32 levels in quantizers and 32-PAM. Energy was optimally allocated between different channel states. We include the performance of the similar system for the AWGN channel with the same average CNR and the performance of the system with no CSI available at the transmitter for comparison. The following observations can be made based on the results in Fig. 15. The Rayleigh fading channel leads to the need to increase the transmitted power by 2 dB for the informed transmitter case and by about 2.5 dB for the uninformed transmitter case. Channel state information availability does not significantly improve the performance of the jointly designed system. The results in Fig. 15 validate our choice for using a limited number of multiresolution codebooks instead of designing a separate codebook for each particular channel state.

It is interesting to recognize that the problem we have just formulated is essentially the same as the problem of optimal "loading" for the multicarrier systems [26]. Multicarrier communication systems explore frequency division multiplexing to create a large number of orthogonal channels. In many cases, each channel can be characterized as an AWGN channel with known (but different) noise variance. The total transmission energy has to be optimally distributed between the carriers to maximize the throughput of the system. The theoretical solution to this problem (the so-called "inverse water pouring" algorithm is derived for a specific case of Gaussian source pdf) can be found in [13]. Unfortunately, direct application of this result to practical systems is not possible due to different source statistics, granularity of channel rates, and other constraints. The common approach to loading is the application of iterative energy/rate allocation based on marginal return techniques. Alternatively, the energy allocation problem can be solved using the Lagrange multiplier technique as explained above [and illustrated in Fig. 14(b)]. Details may be found in [27].

It is also possible to easily extend this to the case of a composite source consisting of L i.i.d. sources $\{X_i\}_{i=0}^{L-1}$ similar to the treatment of Section IV. We assume that the transmission time is much longer than the durations of fades. Then, data from each source component X_i will be transmitted in all possible channel states $\{s_j\}_{j=0}^{N-1}$, and all possible source/channel combinations (X_i, s_j) will be encountered with some known probabilities $p_{i,j}$ ($p_{i,j}$ is the probability of a particular state $j - p_j$ times the probability of the ith source component). The optimal power allocation is performed among $L \times N$ such combinations using the constant slope policy for E-D curves, which is similar to the single source case in Fig. 14(b).

VI. Discussion

We have introduced a natural and efficient multiresolution-based framework to do joint source-channel coding and formulated an efficient Lagrangian-based optimization algorithm to accomplish this. Our basic idea is derived from the simple concept of optimally matching the source resolution to the channel "resolution." That is, having a multiresolution description of both source codes (source "quality" levels) *and* channel codes (degrees of channel noise immunity provided) enables the efficient matching of these resolution "trees" to each other and to the instantaneous state of the channel. By having an multiresolution "menu" of resolutions, both encoder and decoder have the flexibility of "picking off" the levels that are optimal for a given channel condition. If both transmitter and receiver are informed of the CSI, they can both adapt to it. If the channel gets "bad," the source resolution is lowered because operating at full resolution does more harm than good. As the channel gets "good," the resolution can be increased. If the transmitter is uninformed of the CSI, the multiresolution framework is still attractive as the transmitter "broadcasts" a multiresolution description, with the receiver picking the resolution that is most appropriate for the given channel state.

In this work, we have quantified how these different resolution "codebooks" for both source (using a subband decomposition) and channel (accomplished through the novel idea of embedded constellations) should be designed for slow time-varying channels and, further, how the optimal resolution-changing strategies should be incorporated. The proposed adaptive multiresolution paradigm transcends the details of algorithmic, architectural, and implementational aspects of particular communications systems. In our particular system, we show that 2–3 dB of gain in SNR are typically realizable by invoking a multiresolution-based JSCC approach over source-channel optimized single-resolution designs based on standard modulation constellations (i.e., that exclude the modem explicitly from the optimization loop).

An interesting direction, which we plan to explore in the future, is to consider additional constraints on the maximum allowed total number of the reconstruction levels (this addresses the storage requirements) in the receiver and to investigate the optimal design in this case to explore tradeoffs of performance versus storage complexity. Other extensions include the integration of the error control coding in our multiresolution framework. As mentioned in Section II, the embedding in this work is solely achieved through the uncoded modulation constellations. We expect a more efficient design if channel coding is combined with our proposed multiresolution modulation in a jointly optimized scheme.

It is insightful to note, in the context of image transmission applications, that one aspect of our framework applied to the PAM case requires the transmission of as many coefficients as in the original image, i.e., an $N \times N$ image will require the transmission of N^2 wavelet coefficients. This is potentially wasteful but unavoidable due to the i.i.d. source assumption invoked. Recent advances in wavelet-based image coding reveal that this assumption is far from accurate, as evinced by the success of coders based on data structures that exploit the dependencies among the wavelet coefficients (e.g., based on zero trees, morphology, classification-based approaches, etc., [28]–[30]). It can be noted that the methodology advocated in this work can be appropriately modified to include such dependencies, at least partially. For example, using a zero-tree data structure [28], [31], it is possible to reduce the number of coefficients needing to be transmitted by grouping sets of insignificant coefficients together as special zero-tree symbols (which, however, need to be transmitted "losslessly" using conventional digital transmission techniques.) In [32], we show how the PAM transmission scheme proposed here combined optimally with a conventional unequal error-protected coded scheme leads to significant gains in quality for the transmission of still images over time-varying channels. Further investigations of this framework are currently under way.

APPENDIX

Here, we derive encoding/decoding rules for the source coder in our example in Section II-D-1 with the number of levels $N = 4$ and number of different reconstruction codebooks $M = 2$. The derivations for the general case of Section III may be performed similarly.

Given the cost function

$$J = \sum_{s=0}^{M-1} p_s \sum_{i=0}^{N-1} \int_{\gamma_i} f(x) \sum_{j=0}^{N-1} (x - c_{s,j})^2 \cdot \text{Prob}_s(j|i)\, dx + \lambda \sum_{i=0}^{N-1} m_i^2 \int_{\gamma_i} f(x)\, dx. \quad (14)$$

By setting the partial derivative of J with respect to $c_{s,j}$ to zero, we obtain

$$\frac{\partial J}{\partial c_{s,j}} = 0 = -p_s \sum_{i=0}^{N-1} \int_{\gamma_i} f(x)\, 2\, c_{s,j}\, (x - c_{s,j}) \cdot \text{Prob}_s(j|i)\, dx.$$

Therefore, the necessary condition (which is known as a weighted centroid condition [7]) for the minimum is

$$c_{s,j} = \frac{\sum_{i=0}^{N-1} \text{Prob}_s(j|i) \int_{\gamma_i} f(x)\, x dx}{\sum_{i=0}^{N-1} \text{Prob}_s(j|i) \int_{\gamma_i} f(x)\, dx}.$$

It can be shown that a sufficient condition for the minimum is always satisfied for the cost-function above.

To obtain the encoding rule, we have to find the source partitioning that minimizes the expected value of a cost function (14) for each value of $X = x$, i.e.,

$$i = \arg \min_l [\mathcal{E}\{J | x \in \gamma_l\}].$$

Taking the expectation of the cost function (14) conditioned on $x \in \gamma_l$, we obtain

$$\mathcal{E}\{J | x \in \gamma_l\} = \int_{\gamma_l} \left[\lambda (m_l)^2 + \sum_{s=0}^{M-1} p_s \sum_{j=0}^{N-1} \cdot \text{Prob}_s(j|l)\, (x - c_{s,j})^2 \right] f(x)\, dx.$$

The expected value of the cost function achieves the minimum when the expression in the square brackets is minimized. Then, the mapping rule

Assign x to γ_i if:

$$\cdot\, i = \arg \min_l \cdot \left[\lambda (m_l)^2 + \sum_{s=0}^{M-1} p_s \sum_{j=0}^{N-1} \text{Prob}_s(j|l)\, (x - c_{s,j})^2 \right].$$

is optimal.

REFERENCES

[1] K. Ramchandran, A. Ortega, K. M. Uz, and M. Vetterli, "Multiresolution broadcast for digital HDTV using joint source-channel coding," *IEEE J. Select. Areas Commun.,* vol. 11, pp. 6–23, Jan. 1993.

[2] N. Tanabe and N. Farvardin, "Subband image coding using entropy-coded quantization over noisy channels," *IEEE J. Select. Areas Commun.,* vol. 10, pp. 926–943, June 1992.

[3] K. Fazel and M. Ruf, "Combined multilevel coding and multiresolution modulation," in *Proc. IEEE Int. Conf. Commun.,* Geneva, Switzerland, May 1993, pp. 1081–1085.

[4] M. Polley, S. Wee, and W. Schreiber, "Hybrid channel coding for multiresolution HDTV terrestrial broadcasting," in *Proc. IEEE Int. Conf. Image Process.,* Nov. 1994, pp. 243–247.

[5] T. Cover, "Broadcast channels," *IEEE Trans. Inform. Theory,* vol. IT-18, pp. 2–14, Jan. 1972.

[6] M. Vetterli and J. Kovacevic, *Wavelets and Subband Coding.* Englewood Cliffs, NJ: Prentice-Hall, 1995.

[7] A. Gersho and R. M. Gray, *Vector Quantization and Signal Compression.* Norwell, MA: Kluwer, 1992.

[8] H. Kumazawa, M. Kasahara, and T. Namekawa, "A construction of vector quantizers for noisy channels," *Electron. Eng. Japan,* vol. 67-B, pp. 39–47, 1984.

[9] N. Farvardin and V. Vaishampayan, "Optimal quantizer design for noisy channels: An approach to combined source-channel coding," *IEEE Trans. Inform. Theory,* vol. IT-33, pp. 827–838, Nov. 1987.

[10] H. S. Wang and N. Moayeri, "Optimal quantization for a finite-state noisy channel," in *Abstr. IEEE Int. Symp. Inform. Theory,* Jan. 1990, p. 79.

[11] T. Duman and M. Salehi, "Optimal quantization for finite-state channels," *IEEE Trans. Inform. Theory,* vol. 43, pp. 758–765, Mar. 1997.

[12] T. Berger, *Rate Distortion Theory.* Englewood Cliffs, NJ: Prentice-Hall, 1971.

[13] T. M. Cover and J. A. Thomas, *Elements of Information Theory.* New York: Wiley, 1991.

[14] F. H. Liu, P. Ho, and V. Cuperman, "Joint source and channel coding using a nonlinear receiver," in *Proc. IEEE Int. Conf. Commun.,* June 1993, pp. 1502–1506.

[15] H. S. Wang and N. Moayeri, "Finite-state Markov channel–A useful model for radio communication channels," *IEEE Trans. Veh. Technol.,* vol. 44, pp. 163–171, Feb. 1995.

[16] S. Redl, M. Weber, and M. W. Oliphant, *An Introduction to GSM.* Norwell, MA: Artech House, 1995.

[17] Z. Zhang and T. Berger, "New results in binary multiple descriptions," *IEEE Trans. Inform. Theory,* vol. 33, pp. 502–521, July 1987.

[18] W. H. Equitz and T. M. Cover, "Successive refinement of information," *IEEE Trans. Inform. Theory,* vol. 37, pp. 269–275, Mar. 1991.

[19] V. Vaishampayan, "Design of multiple description scalar quantizers," *IEEE Trans. Inform. Theory,* vol. 39, pp. 821–834, May 1993.

[20] S. Mallat, "A theory for multiresolution signal decomposition: The wavelet representation," *IEEE Trans. Pattern Anal. Mach. Intell.,* vol. 11, pp. 674–693, July 1989.

[21] K. A. Birney and T. R. Fisher, "On the modeling of DCT and subband image data for compression," *IEEE Trans. Image Processing,* vol. 4, pp. 186–193, Feb. 1995.

[22] M. Ruf and J. Modestino, "Rate-distortion performance for joint source and channel coding of images," in *Proc. IEEE Int. Conf. Image Process.,* 1995, pp. 77–80.

[23] Y. Shoham and A. Gersho, "Efficient bit allocation for an arbitrary set of quantizers," *IEEE Trans. Acoust., Speech, Signal Processing,* vol. 36, pp. 1445–1453, Sept. 1988.

[24] P. Chou, T. Lookabaugh, and R. Gray, "Entropy-constrained vector quantization," *IEEE Trans. Acoust., Speech, Signal Processing,* vol. 37, pp. 31–42, Jan. 1989.

[25] K. Ramchandran and M. Vetterli, "Best wavelet packet bases in a rate-distortion sense," *IEEE Trans. Image Processing,* vol. 2, pp. 160–175, Apr. 1993.

[26] P. S. Chow, "Bandwidth optimized digital transmission techniques for spectrally shaped channels with impulse noise," Ph.D. dissertation, Stanford Univ., Palo Alto, CA, 1993.

[27] S. Pradhan and K. Ramchandran, "Efficient layered video delivery over multicarrier systems using optimised embedded modulations," in *Proc. IEEE Int. Conf. Imag Process.,* 1997, vol. 3, pp. 452–455.

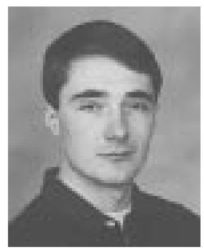
[28] J. M. Shapiro, "Embedded image coding using zerotrees of wavelet coefficients," *IEEE Trans. Signal Processing,* vol. 41, pp. 3445–3462, Dec. 1993.

[29] S. Servetto, K. Ramchandran, and M. Orchard, "Image coding based on a morphological representation of wavelet data," submitted for publication.

[30] R. L. Joshi, T. R. Fischer, and R. H. Bamberger, "Optimum classification in subband coding of images," in *Proc. IEEE Int. Conf. Image Processing,* Austin, TX, Nov. 1994, pp. 883–887.

[31] Z. Xiong, K. Ramchandran, and M. T. Orchard, "Space-frequency quantization for wavelet image coding," *IEEE Trans. Image Processing,* vol. 6, pp. 677–693, May 1997.

[32] I. Kozintsev and K. Ramchandran, "A hybrid compressed-uncompressed framework for wireless image transmission," in *Proc. IEEE Int. Conf. Image Process.,* 1997, vol. 2, pp. 77–80.



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