# MULTIRESOLUTION JOINT SOURCE-CHANNEL CODING USING EMBEDDED CONSTELLATIONS FOR POWER-CONSTRAINED TIME-VARYING CHANNELS. *

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## ABSTRACT

We explore joint source-channel coding (JSCC) for time-varying (slow-fading Rayleigh) channels, using a multiresolution (MR) framework for both source coding and transmission (via a novel MR modulation constellation). We tackle the important case of informed receiver but uninformed transmitter, i.e. where the receiver has access to the channel state information (CSI), but the transmitter does not. We describe an algorithm which jointly optimizes the design of the MR source codebook, the MR constellation, and the decoding strategy of optimally matching the source and signal constellation resolution trees according to the time-varying channel, and show how this leads to improved performance over separately designed source and channel coders.

## 1. INTRODUCTION AND BACKGROUND

The benefits of multiresolution (MR) joint source-channel coding (JSCC) for digital broadcast were established recently in [1]. The basic idea there is derived from Cover's classic result [2] that in multiuser communications, where a single source transmits information to two (or more) receivers of different fidelity, joint information transfer can be maximized if the transmitter superimposes the information meant for the strong receiver in the information meant for the weaker one, see Fig.1. This endorses the use of a MR framework, where both receivers have access to the "coarse" information, while enabling the stronger receiver to extract the underlying "detail" information as well. In [1], this idea was applied to the design of modulation constellations: see Fig.2 for an example of 2-level MR constellations, which can be seen to contain "clouds" of "satellites," characterized by an intracloud to intercloud distance ratio $\mu$. Note that two levels of unequal noise immunity are offered by these constellations, as represented by the satellites and the clouds in which they are embedded. This MR transmission scheme can be matched to an MR source-coding scheme (e.g. using hierarchical subband coding) where the coarse (important) source layer maps to the clouds, and the detail (refinement) layer maps to satellites within each cloud.

In this work, we describe how the use of MR constellations for point-to-point time-varying channels can lead to similar gains as the broadcast channel. The analogy is not surprising since both cases refer to multi-channel environments. In seeking a MR environment, we are motivated by the *increasing need to be flexible and scalable* in source and channel coding architectures due to increased interconnectivity and heterogeneity. We consider a slow-fading Rayleigh channel, a popular model in wireless communications, with interleaving being used to render the channel memoryless. In this work, we tackle the case where the channel state information (CSI) indicating the instantaneous channel Carrier-to-Noise ratio (CNR)[1] is available only at the receiver, as applicable when there is no feedback channel from the receiver to the transmitter. This is the case of the "informed receiver" studied in [3].

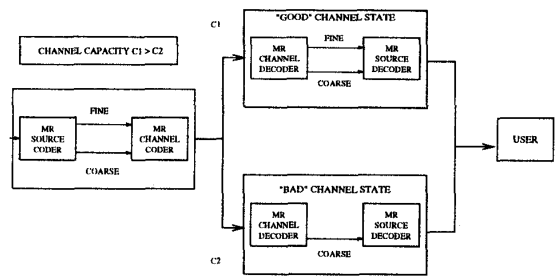


**Figure 1. Block diagram of 2-resolution joint source channel coding scheme.**

In motivating our work, we start with a brief overview. We begin by noting that optimal source coding has been a field of considerable interest and is well-understood, e.g. the Lloyd-Max algorithm for scalar quantizer design and the Generalized Lloyd Algorithm (GLA) for vector quantizer design [4]. These algorithms optimize the source codebook design using essentially alternate iterative optimizations of the encoder (with the decoder being fixed) and the decoder (with the encoder being fixed) and assuming a noiseless transmission channel. This design methodology has been extended to the case of noisy channels by several researchers (e.g. [5, 6, 7] and others) assuming channel models like the Binary-Symmetric-Channel or more general multi-alphabet channels with a given (i.e. *fixed*) transition probability matrix relating the channel input/output alphabets. This raises the interesting question of whether further gains can be obtained by having the freedom to optimally alter these symbol transition error probabilities (by including the modulation constellation in the loop) and judiciously trading off some error probabilities for others *while keeping the transmitted constellation power fixed.* This leads to the problem of jointly designing the source codebook and the channel constellation within the constraints of fixed transmission power in order to reduce the overall end-to-end distortion. This has been addressed recently in [8], where a single resolution design was used, the transmitter was assumed to be informed of the CSI, and the optimized modulation constellation was unstructured. In this paper, in ad-

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[1] Note that we will use CNR to refer to the channel Carrier-to-Noise ratio, while we will use SSNR to refer to the source Signal-to-Noise ratio.

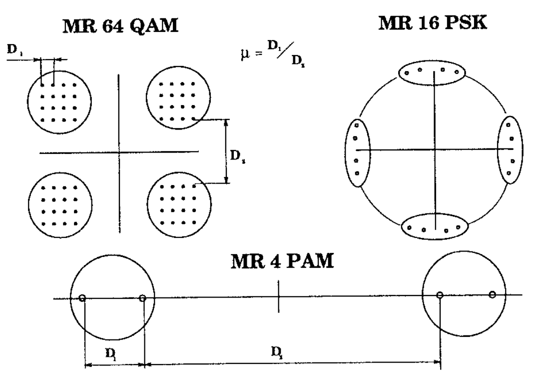


**Figure 2. Some MR constellations where $\mu$ parametrizes the intracloud to intercloud ratio.**

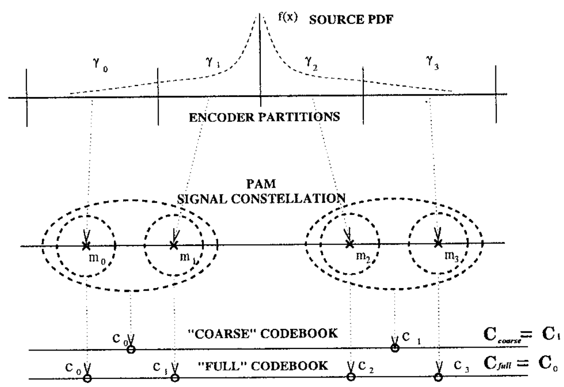


**Figure 3. Two resolution JSCC scheme for MR 4-PAM and source codebooks having 2/4 levels.**

dition to tackling the JSCC problem in a more intuitively appealing framework, we depart in four significant regards: **(i)** we use a *multiresolution approach* advocating the use of multiple[2] codebook resolutions; **(ii)** we assume an *uninformed transmitter* case, thereby stripping the transmitter of the capability to adapt to the time-varying channel conditions; **(iii)** in the interests of practicality, we impose a *regular structure* on the resulting optimized constellation, using the intuitive idea of clouds and satellites (see Fig. 2 and Fig. 3); and **(iv)** we use a multistate AWGN channel model that can handle any types of CNR distribution for slow-varying channels. In the interests of simplicity, we confine ourselves to a scalar quantization framework, though extensions to vector quantization can be made.

The main contribution of this paper is that it tackles the JSCC problem for time-varying channels in a MR setting using MR source codebooks and embedded channel constellation. We show how the optimal design strategy dictates for *the receiver to adjust its source codebook resolution according to the CSI*, and describe how to find, for a given time-varying channel, the *critical* CNR thresholds at which such resolution transitions should occur.

## 2. PROPOSED FRAMEWORK

Our framework is easiest explained through an illustrative example of a two-state AWGN channel model. This is insightful as it can be easily generalized to the desired Rayleigh channel case by considering an N-state AWGN channel, as N gets sufficiently large. Let $h^2 = E/N_0$ be the received signal-to-noise power ratio. Then in a Rayleigh model with parameter $\hat{h}$ the p.d.f. of $h$ is $f(h) = 2h/\hat{h}\ exp(-h^2/\hat{h}^2)$. By approximating this distribution by a piecewise constant p.d.f. having $N$ regions we achieve a multistate AWGN representation which becomes exact as $N$ becomes large. Note that this approximation can be made for other CNR distributions, as well.

### 2.1. Example of 4-level quantizer, 4 PAM, and 2-state AWGN channel

Suppose the channel can be in one of only two different states. In each state, the channel is AWGN with a different noise variance, with the states being labeled "good" and "bad." Recall that the receiver knows the actual channel state while the transmitter has knowledge of only long-term channel statistics, i.e. the state probabilities. Suppose we want to transmit an i.i.d. (scalar) source $x$ with p.d.f. $f(x)$ quantized to 4 levels, across this channel using a 4-PAM modulation constellation, assuming a one-to-one mapping between the encoder-partitioning $\{\gamma_i\}_{i=0}^3$ and the constellation points $\{m_i\}_{i=0}^3$, as shown in Fig. 3. The joint encoder/modulator operation is therefore to partition the source $x$ into intervals $\{\gamma_i\}_{i=0}^3$ and map each $\gamma_i$ to the corresponding constellation point $m_i$. At the receiver, suppose we use a hard-decision demodulator which declares, based on optimized demodulator thresholds $\{t_i\}$, which $m_i$ was transmitted. Finally, the decoder performs a one-to-one mapping between the $m_i$'s and the source reconstruction codewords $c_i$. Given this framework, the question is how to design the end-to-end system in order to minimize the overall distortion (due to source quantization *and* channel noise) for a fixed average transmission power. The system parameters include:
(i) the source encoder partitions $\Gamma = \{\gamma_i\}$;
(ii) the channel modulation constellation $\mathcal{M}=\{m_i\}$;
(iii) the hard-decision receiver thresholds $\mathcal{T} = \{t_i\}$;
(iv) the source decoder codebook $\mathcal{C}$ containing the reconstruction codewords $\{c_i\}$.

It is clear that the optimal solution to our problem would be one that allows two *different* distortion-optimal decoders for each channel state. The receiver should use the CSI to switch between these two decoders. Such a design, while manageable for a two-state AWGN channel model, is clearly impractical when the number of states gets large (as needed to approximate the desired Rayleigh channel model) since this would require a separate design for each channel state. An obvious way of alleviating this problem is to allow only a few (MR) codebooks, and devise an optimal decoding strategy for deciding which codebook should be used in each channel state. The approach can be easily generalized to any desired N-state model. For a 2-resolution case, the decoder has two reconstruction codebooks. In our example see Fig.3, the full-resolution codebook $\mathcal{C}_0 = \mathcal{C}_{full}$ has 4 reconstruction levels while the coarse-resolution codebook $\mathcal{C}_1 = \mathcal{C}_{coarse}$ has only 2 reconstruction levels[3].

While our framework supports the overlaying of forward error correction codes, in this work we restrict ourselves, w.l.o.g., to achieving noise immunity solely through the idea of constellation clouds (see Fig.2). We impose this intuitively appealing regular structure on our modulation constellation, by characterizing it by the set of parameters $\{\mu_i\}$ that reflect the difference in noise immunity between consecutive layers. For a 2-layer model of 4-PAM example in Fig.3 only one parameter $\mu$ is needed. The two-resolution choice therefore offers a tradeoff between source coder precision and channel noise immunity. In our example, the

[2] Without loss of generality, we consider a 2-resolution case.

[3] We ignore the trivial zero-resolution codebook having one entry, i.e. the mean of the input distribution.

choice of the full-resolution (4-level) codebook $\mathcal{C}_0$ and its associated full 4-point constellation is biased towards increased source (and decreased channel) resolution, while the opposite is true for the coarse-resolution (2-level) codebook choice $\mathcal{C}_1$ and its associated 2-"cloud" constellation representation. The receiver should devise a (predesigned) rule that assigns the optimal codebook choice to each of the two channel states $s = 0, 1$ (and in general for each of N states $s = 0, \ldots N-1$) from among $\mathcal{C}_0$ and $\mathcal{C}_1$ (and in general - $\mathcal{C}_0, \mathcal{C}_1, \ldots, \mathcal{C}_{L-1}$, where $L$ is the number of resolutions). We call this rule a *strategy* **S**. In our example, then, there are three possible strategies:

(i) $\mathbf{S}_0 = (\mathcal{C}_0, \mathcal{C}_0)$, i.e. full resolution codebook is used in "good" and "bad" states;
(ii) $\mathbf{S}_1 = (\mathcal{C}_0, \mathcal{C}_1)$, i.e. full resolution codebook is used in "good" state and coarse resolution in "bad" state;
(iii) $\mathbf{S}_2 = (\mathcal{C}_1, \mathcal{C}_1)$, i.e. coarse resolution codebook is used in "good" and "bad" states (which is a subcase of $S_0$ but we keep it for generality).

We need to try all possible strategies and pick the best one. This is easily extended to the N-channel case, where imposing the "monotonicity" requirement that the optimal strategy at a better (i.e. less noisy) channel state would necessarily use the same or better resolution codebook choice, can result in dramatically reduced search complexity for the optimal **S**.

As motivated earlier, since the multistate AWGN channel model, with a sufficiently large number of states, can approximate the Rayleigh channel model (and others) with arbitrary fidelity, we turn our attention to its analysis.

## 3. PROBLEM STATEMENT FOR MULTISTATE AWGN CHANNEL.

Let, as before, $\mathcal{M} = \{m_i\}$ refer to the MR constellation set with constellation points $\{m_i\}$ and parameter $\mu$ (see Fig. 2). Let $\Gamma = \{\gamma_i\}$ refer to the encoder partitions, where $\gamma_i$ is assumed to be mapped one-to-one to constellation point $m_i$. Since perfect CSI is available at the receiver, it can optimize the choice of demodulator thresholds $t_i$ as well as the (predesigned) multistate "resolution" strategy **S**.

As in the two-resolution, two-state AWGN example, the strategy **S** for an L-resolution, N-state AWGN case can be written as an N-vector whose entries are selected from among $\mathcal{C}_0, \mathcal{C}_1, \ldots, \mathcal{C}_{L-1}$, where each entry corresponds to the particular resolution picked for the particular channel state. The problem is to find, for a target average transmission power $P_{av}$ the optimal choices for $\mathcal{M}$, $\Gamma$ and **S**, such that the average end-to-end expected distortion (we will use MSE) $D(X, \hat{X})$, which is a weighted average of the conditional distortions given the channel states, between the input $X$ (assumed to be a scalar random variable having a p.d.f of $f(x)$) and its estimated approximation $\hat{X}$, is minimized:

$$\min_{\mathcal{M},\Gamma,\mathbf{S}} [D(X,\hat{X}) = \sum_s p_s \; D(X,\hat{X}|\mathbf{S})], \qquad (1)$$

subject to the fixed power constraint:

$$P_{av} = \sum_i m_i^2 \int_{\gamma_i} f(x)\, dx \qquad (2)$$

where $p_s$ is the probability of the channel being in the $s$-th state.

## 4. PROBLEM SOLUTION.

The constrained problem of (1) can be converted into an unconstrained one using the Lagrangian multiplier method using the intermediate variable $\lambda \geq 0$, which trades off distortion for power. That is, the equivalent unconstrained problem is to minimize $J = D + \lambda P$. The solution for a fixed value of $\lambda$ and $\mu$ can be found by iterative procedure similar to the GLA [4]. Then by using a convex search on $\lambda$ for a finite set $\mu$'s the optimal design can be obtained[4]. Formally, this is summarized in the following algorithm:

### 4.1. JSCC ALGORITHM

**Step 0** Initialize strategy **S** defining which of the $L$ multiresolution codebooks (from among $\{C_i\}_{i=1}^{L}$) is assigned to each of the $N$ channel states $\{s_i\}_{i=1}^{N}$.

**Step 1** Initialize the value of the Lagrange multiplier $\lambda$.

**Step 2** Initialize the constellation parameter $\mu$.

**Step 3** Initialize the encoder partitions $\Gamma$, decoder reconstruction codebooks $\mathcal{C}_r$ for each resolution $r$, receiver thresholds $\mathcal{T}_s$ for each channel state $s$, and modulation constellation $\mathcal{M}$ .

**Step 4** For a fixed $\mathcal{M}$, apply a JSCC-modified version of the Lloyd-Max algorithm to iteratively optimize the encoder partitions $\gamma_i$ and the decoder reconstruction levels $\mathcal{C}_r$ together with optimization of demodulator thresholds $\mathcal{T}_s$ for each channel state $s$, so that the cost function is minimized. For our model the distortion can be written as:

$$D = \sum_r \sum_{s\in S_r} p_s [\sum_i \int_{\gamma_i} f(x) \sum_j (x - c_{r,j})^2 P_s(j|i)\, dx],$$

where $r$ is the resolution index, $S_r$ is the set of states for which the codebook $\mathcal{C}_r$ with codewords $c_{r,j}$ is used (this is determined by the strategy **S**) and $P_s(j|i)$ is the *a priori* probability of receiving $m_j$ given that $m_i$ is sent. The expression in brackets is the distortion in a particular state $s$, and we average it over all possible resolutions $r$ and channel states $s \in S_r$ mapping to these resolutions. Note that the transitional probabilities $P_s(j|i)$ depend on the average channel $CNR_{av}$ in state $s$ (this is a function of the assumed fading model) and also on the thresholds $\mathcal{T}_s$.

The equations for iterative optimization can be easily obtained by setting the partial derivatives of the cost function w.r.t. the respective parameters equal to zero. Details are omitted. The following two-step iterative process results:

(a) Optimize the reconstruction codebooks $\mathcal{C}_r$ for a fixed encoder partitioning $\Gamma$ using a weighted centroid condition (WCC). Then for each codeword at resolution $r$ we can write:

$$c_{r,j} = \sum_{s\in S_r} p_s [\sum_i \alpha(i|j)\beta_i] \qquad (3)$$

where $\beta_i$ are the (noiseless channel) centroids, and $\alpha(i|j)$ are the *a posteriori* probabilities of receiving $c_{r,j}$ given that $x \in \gamma_i$ .

(b) Assign source input $x$ to the nearest encoder partition $\gamma_i$ in the Lagrangian sense:

$$\lambda(m_i)^2 + \sum_r \sum_{s\in S_r} p_s \sum_j (P_s(j|i)(x - c_{r,j})^2) \quad (4)$$

At each step a separate optimization for receiver thresholds $\mathcal{T}_s$ is performed to obtain the best performance of the hard decision receiver, i.e. to minimize the value of the cost function given that other parameters are fixed. This optimization is implicitly performed in the calculation of the transitional probabilities $P_s(j|i)$.

**Step 5** Do the iterations of **Step 4** till local convergence.

**Step 6** Sweep Steps 3-5 over a (fine) discrete set of values of the constellation $\mu$ and pick the value of $\mu^*$ for which the cost function is minimum.

[4] In the case of multiple constellation parameters $\{\mu_i\}$, a gradient search can be used.

**Step 7** Do a convex search over $\lambda \geq 0$ (Steps 2-7) to find the optimal $\lambda^*$ that satisfies $P_{av}$.

**Step 8** Search all candidate strategies and pick the best.

Note that the search for the best strategy **Step 8** does not have to be exhaustive but is instead a fast convex search due to the monotonic relationship between states and resolutions.

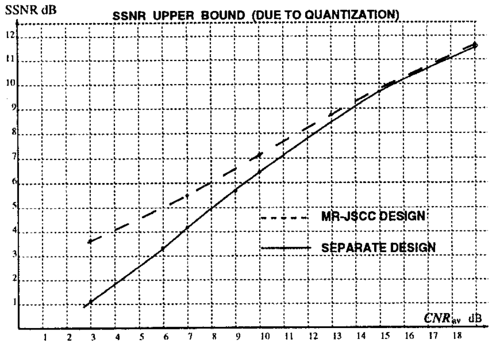


**Figure 4. Performances of designed and reference systems for the case of Rayleigh channel, unit variance Laplacian source, PAM-8 and 8/4-level $\mathcal{C}_{full}/\mathcal{C}_{coarse}$. The reference system consists of separately optimized 8-level source quantizer, and 8-PAM equidistant constellation with optimized receiver thresholds, while the MR-JSCC scheme is based on the best strategy for each $CNR_{av}$.**

## 5. SIMULATION RESULTS

We present simulation results for a slow-fading Rayleigh channel model with average CNR denoted by $CNR_{av}$. We assume a unit variance Laplacian source quantized to two resolutions using 8/4-level reconstruction codebooks $\mathcal{C}_{full}/\mathcal{C}_{coarse}$, and an 8-PAM constellation. We approximate the Rayleigh channel by an almost exact 20-state AWGN model.

Fig. 4 shows the results of applying the MR-JSCC algorithm described in section 4.1.. The reference system in Fig. 4 consists of separately optimized 8-level source quantizer and 8-PAM equidistant constellation with optimized receiver thresholds, while the MR-JSCC scheme is based on the best strategy for each $CNR_{av}$. Note how significant gains can be obtained at low $CNR_{av}$'s. The optimal MR-JSCC systems do not have equidistant modulation points, especially at low $CNR_{av}$'s.

Fig. 5 shows the relative frequency of usage of the full and coarse codebooks in the optimal design. As expected the coarse codebook is used more frequently for Rayleigh channels having lower $CNR_{av}$ values. Note that for every channel average $CNR_{av}$ the best performance is achieved for the strategy that uses $\mathcal{C}_{coarse}$ for the percentage of time shown.

Fig. 6 shows the regions of optimal strategies corresponding to different usages of coarse/full resolution codebooks, highlighting the locations of critical points where resolution transitions occur.

## 6. DISCUSSION

Simulation results show that it is possible to obtain gain in system performance by doing a joint source channel coding with embedded constellations. The MR-JSCC framework itself is very general. It is possible, for instance, to use channel coding over embedded JSCC design [1]. Also multidimensional signal constellations offer more flexibility than simple 1D or 2D constellations like PAM or QAM. The basic paradigm involves the design of optimally matched MR

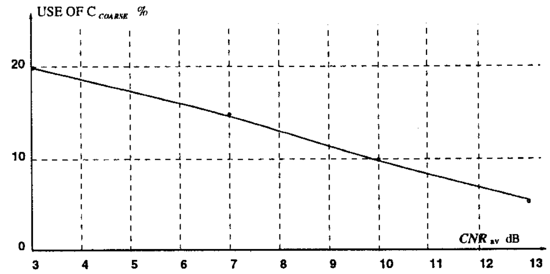


**Figure 5. Frequency of usage of coarse reconstruction codebook $\mathcal{C}_1 = \mathcal{C}_{coarse}$ in the optimal strategy for unit variance Laplacian source and 8-PAM for Rayleigh channel. $\mathcal{C}_0 = \{c_i\}_{i=0}^{7}$, $\mathcal{C}_1 = \{c_i\}_{i=0}^{3}$.**

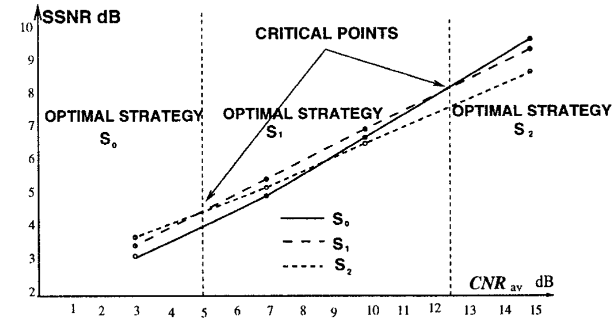


**Figure 6. Regions of optimal strategies bounded by critical $CNR_{av}$ values. $S_0$, $S_1$, $S_2$ use full resolution codebook $\mathcal{C}_0 = \{c_i\}_{i=0}^{7}$ for 100%, 90% and 80% of time respectively. Note the positions of "critical" $CNR_{av}$ points where resolution transitions occur.**

"source" and "channel" trees so that the overall distortion is minimized and all practical constraints are met. We will address these issues in future work.

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